1 a $x \vee x = (x \vee x) \wedge 1$ (Axiom 4)
$= (x \vee x) \wedge (x \vee x')$ (Axiom 5)
$= x \vee (x \wedge x')$ (Axiom 3)
$= x \wedge 1$ (Axiom 5)
$= x$ (Axiom 4)

b $x \wedge x = (x \wedge x) \vee 0$ (Axiom 4)
$= (x \wedge x) \vee (x \wedge x')$ (Axiom 5)
$= x \wedge (x \vee x')$ (Axiom 3)
$= x \wedge 1$ (Axiom 5)
$= x$ (Axiom 4)

c $(x')' = (x')' \vee 0$ (Axiom 4)
$= (x')' \vee (x \wedge x')$ (Axiom 5)
$= ((x')' \vee x) \wedge ((x')' \vee x')$ (Axiom 3)
$= ((x')' \vee x) \wedge 1$ (Axiom 5)
$= ((x')' \vee x) \wedge (x' \vee x)$ (Axiom 5)
$= ((x')' \wedge x') \vee x$ (Axiom 3)
$= 0 \vee x$
$= x$

d proof

e proof

2 $a \vee [(b \wedge c') \wedge (d \wedge b')] = a \vee [b \wedge b' \wedge c' \wedge d]$ (Axioms 1 & 2)
$= a \vee [0 \wedge c \wedge d]$
$= a \vee 0$
$= a$

3 a

x	y	y'	$x \wedge y'$	$f(x,y)$
0	0	1	0	0
0	1	0	0	0
1	0	1	1	1
1	1	0	0	0

b

x	y	z	$x \vee y$	$y \vee z$	$z \vee x$	$f(x,y,z)$
0	0	0	0	0	0	0
0	0	1	0	1	1	0
0	1	0	1	1	0	0
0	1	1	1	1	1	1
1	0	0	1	0	1	0
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

4 **a** $$\begin{aligned}(x \wedge y') \vee (x' \wedge y') &= y' \wedge (x \vee x') \\ &= y' \wedge 1 \\ &= y'\end{aligned}$$

The circuit can be simplified to a y' switch

b $$\begin{aligned}(x \wedge y) \vee (x \wedge y') \vee (x' \wedge y) \vee (x' \wedge y') &= (x \wedge (y \vee y')) \vee (x' \wedge (y \vee y')) \\ &= (x \wedge 1) \vee (x' \wedge 1) \\ &= x \vee x' \\ &= 1\end{aligned}$$

The globe is always on; the circuit can be a single wire with no switches

5 **a** $(x' \wedge y') \vee (x' \wedge y) \vee (x \wedge y)$

b $(x' \wedge y' \wedge z') \vee (x' \wedge y' \wedge z) \vee (x' \wedge y \wedge z) \vee (x \wedge y' \wedge z')$